<u>Sec. 14.3</u>:
Partial Derivatives

What We Will Go Over In Section 14.3

1. What is a Partial Derivative
2. How to Calculate a Partial Derivative
3. Second and Higher Order Partial Derivatives
4. Clairaut’s Theorem

1. What is a Partial Derivative

What is a (Calc 1) Derivative?

Picture of derivative at a point
$f(x) = x^2$ at (3,9)

Picture of derivative function
$f(x) = x^2$ at (3,9)

<u>Notes</u>

- Any time you have a function of 1 variable, you can find the derivative function
- When plugging in a number to the derivative function, you get the slope of the tangent line to the graph of the original function

1. What is a Partial Derivative

Picture/Story of a Calc. 3 Partial Derivative (w.r.t. x)

Given a 2 variable function $f(x,y)$ and a point (a,b,c), slice with $y=b$.

<u>Situation</u>

- You are given a 2-variable function $f(x,y)$ and a point (a,b,c) on its graph
- Slice the surface with the plane $y=b$
- Project the curve to the *xz*-plane
- Find the derivative of this as a function (1-variable since y is constant)
- Plug in $x=a$ to get slope of the tangent line
- When we actually calculate a partial derivative, we will do it slightly differently. This is just the story

1. What is a Partial Derivative

Picture/Story of a Calc. 3 Partial Derivative (w.r.t. y)

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<u>Situation</u>

- You are given a 2-variable function $f(x, y)$ and a point (a,b,c) on its graph
- Slice the surface with the plane $x = a$
- Project the curve to the *yz*-plane
- Find the derivative of this as a function (1-variable since *x* is constant)
- Plug in $y = b$ to get slope of the tangent line
- When we actually calculate a partial derivative, we will do it slightly differently. This is just the story

2. How to Calculate a Partial Derivative

To find the partial derivative of a 2-variable function (w.r.t. *x*)

- Notation: f_x or z_x or $\frac{\partial f}{\partial x}$ or $\frac{\partial z}{\partial x}$
- Find the derivative as a function using Calc. 1 rules, but…

  treat x as the variable and treat all other letters as constants
- The partial derivative as a function will have x's and y's in it
- If a point is given, plug in the x and y coordinate of the point to get the partial derivative at that point

To find the partial derivative of a 2-variable function (w.r.t. *y*)

- Notation: f_y or z_y or $\frac{\partial f}{\partial y}$ or $\frac{\partial z}{\partial y}$
- Find the derivative as a function using Calc. 1 rules, but…

  treat y as the variable and treat all other letters as constants
- The partial derivative as a function will have x's and y's in it
- If a point is given, plug in the x and y coordinate of the point to get the partial derivative at that point

2. How to Calculate a Partial Derivative

Ex 1: If $f(x,y) = x^3 + x^2y^3 - 2y^2$, find $f_x(2,1)$ and $f_y(2,1)$ and draw pictures

2. How to Calculate a Partial Derivative

Ex 2: If $f(x, y) = 4 - x^2 - 2y^2$, find $f_x(1,1)$ and $f_y(1,1)$

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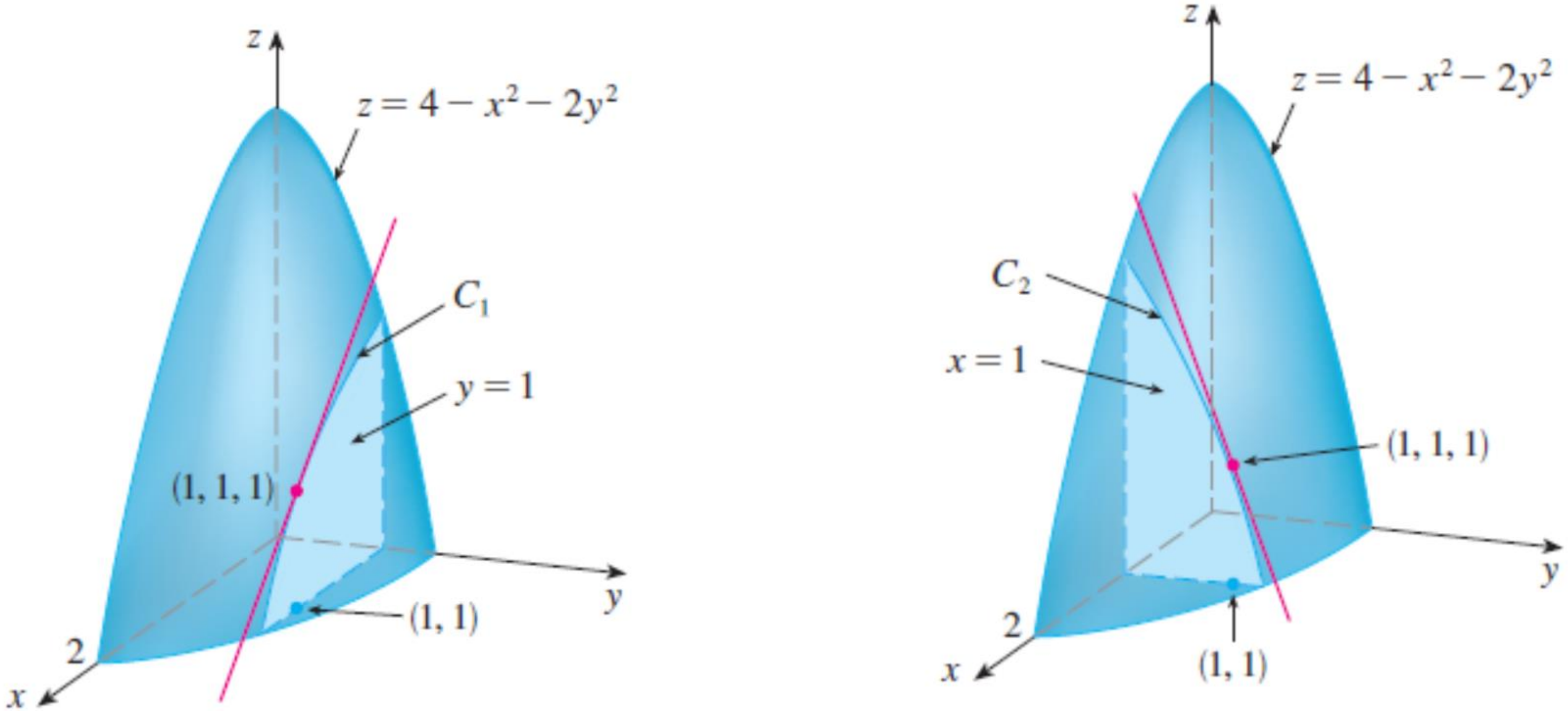

2. How to Calculate a Partial Derivative

Ex 4: If $f(x, y) = \sin\left(\frac{x}{1+y}\right)$, calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

2. How to Calculate a Partial Derivative

<u>Calc. 1 Implicit Differentiation Story…</u>

<u>Recall Calc. 1 CHAIN RULE</u>

<u>Ex</u>: Find…

$[\sin x]'$ $\quad$ $[\sin(x^2+1)]'$ $\quad$ $[\sin y]'$

$[\ln x]'$ $\quad$ $[\ln(e^x+\sin x)]'$ $\quad$ $[\ln y]'$

2. How to Calculate a Partial Derivative

Calc. 1 Implicit Differentiation Story…

Recall Calc. 1 CHAIN RULE

Ex: Find…

$$\frac{d}{dx} e^x \qquad \frac{d}{dx} e^{\sin x} \qquad \frac{d}{dx} e^x$$

$$\frac{d}{dx} x^3 \qquad \frac{d}{dx} \left(x^2 + e^x\right)^3 \qquad \frac{d}{dx} y^3$$

2. How to Calculate a Partial Derivative

Calc. 1 Implicit Differentiation Story…

Ex: Find $y'(4,3)$ if $x^2 + y^2 = 25.$ Picture/Solve for y

2. How to Calculate a Partial Derivative

Calc. 1 Implicit Differentiation Story…

Ex: Find $y'(4,3)$ if $x^2 + y^2 = 25.$ Picture/DO NOT SOLVE FOR *y*

2. How to Calculate a Partial Derivative

Calc. 1 Implicit Differentiation Story…

Most interesting when you can't solve for y…

Ex: Find $\left.\frac{dy}{dx}\right|_{(3,3)}$ if $x^3 + y^3 = 6xy$. Picture

2. How to Calculate a Partial Derivative

How does Implicit Differentiation Change in Calc. 3?

For a 2-variable function, z will be a function of x and y and so you will take partial derivatives instead of regular Calc. 1 derivatives.

2. How to Calculate a Partial Derivative

Ex 5: Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if z is defined implicitly as a function of x and y by the equation

$$x^3 + y^3 + z^3 + 6xyz = 1$$

2. How to Calculate a Partial Derivative

Functions of 3 or more variables … What Changes?

To find the partial derivative of a 3 or more variable function

- Lose the ability to graph
- Whichever letter is the variable, ALL OTHER LETTERS should be treated as constants

2. How to Calculate a Partial Derivative

Ex 6: Find f_x , f_y , and f_z if $f(x, y, z) = e^{xy} \ln z$

3. Second and Higher Order Derivatives

Ex 7: Find the second partial derivatives of $f(x, y) = x^3 + x^2y^3 - 2y^2$

3. Second and Higher Order Derivatives

Ex 8: Calculate f_{xxyz} if $f(x, y, z) = \sin(3x + yz)$

4. Clairaut's Theorem

Clairaut's Theorem

Suppose f is defined on a disk D that contains the point (a, b).

If the functions f_{xy} and f_{yx} are both continuous on D, then

$$f_{xy}(a, b) = f_{yx}(a, b)$$